Mathematical Modelling in the Steel Industry

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1.Background

There is a long tradition of collaboration between university researchers in mathematics departments and research scientists from steel companies. Perhaps the most obvious evidence of this is the number of problems that have been brought to workshops for mathematics-in-industry by steel companies over several decades. Examples of where workshop reports exist include

Flux Consumption in Continuous Casting
Control of Ingot Cooling
Heat and Mass Transfer in Blast Furnaces
Reacting Flow in a Catalyst
Hot Rolling Mechanics
Models for Electromagnetic Stirring
Bulging in Continuous Casting
Mechanics of Cold Rolling
Friction Welding
Impedance Imaging
Roll Press Nip Dynamics
Wire Cooling
Dynamics of mining excavators
Tapered Furnace Feeders
B.O.S Lance Wear
Fastnet Pellet Reduction
Dragline Modelling
Stratification in Ladles
Cooling of Coiled Strip
Buckling of Rolled Strip
Droplet Cooling
Shrinkage in Ingot Solidification
Eutectic Solidification

This list is not comprehensive but it shows that many aspects of steel manufacture, from mining to distribution, are susceptible to mathematical modelling.

●There are two good reasons for the ubiquity of mathematics in the steel industry, and in other large industries:

There are many opportunities for mathematical analogies to be drawn between processes in different areas of application. As an example, the modeling of ingot solidification (Fig.1) is the same as that for a "popsicle" in the food industry (Fig.2).

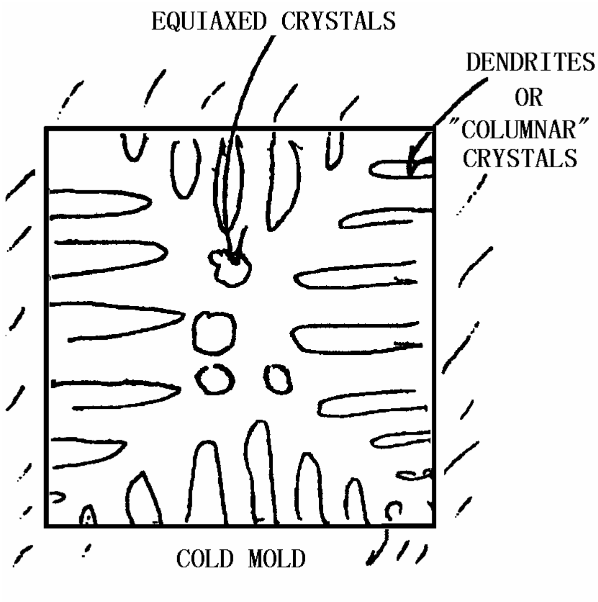

Fig.1 Ingot solidification

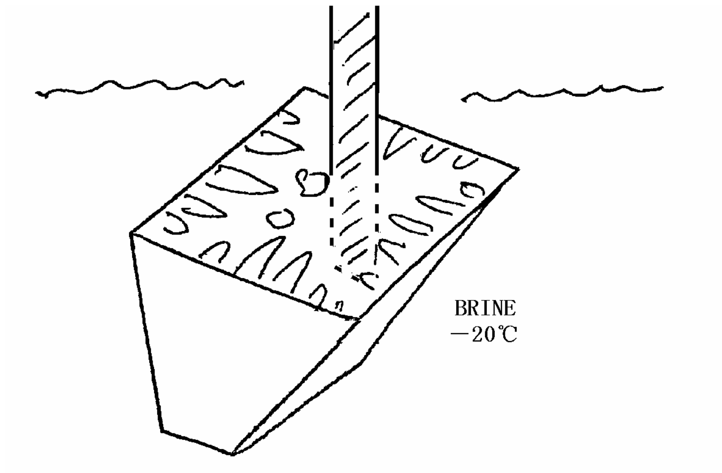

Fig.2 Popsicle solidification

Hence, in this example, all the technology that is known about ingot solidification can be used at once in the food industry.

- The Steel industry has been crucial in stimulating the new mathematical theory of "free boundary problems". This subject did not exist as a coherent branch of mathematics until 1974, but, around that time, modeling processes such as continuous casting made it imperative that the theory of these problems be developed systematically. This theory has been a very fruitful one for mathematics

2.A Problem from BAOSteel: Spray Cooling

This problem and the next have been chosen to show how relatively simple mathematics can provide a scientific framework within which to understand important aspects of steel making. A schematic of spray cooling is shown in Fig.3.

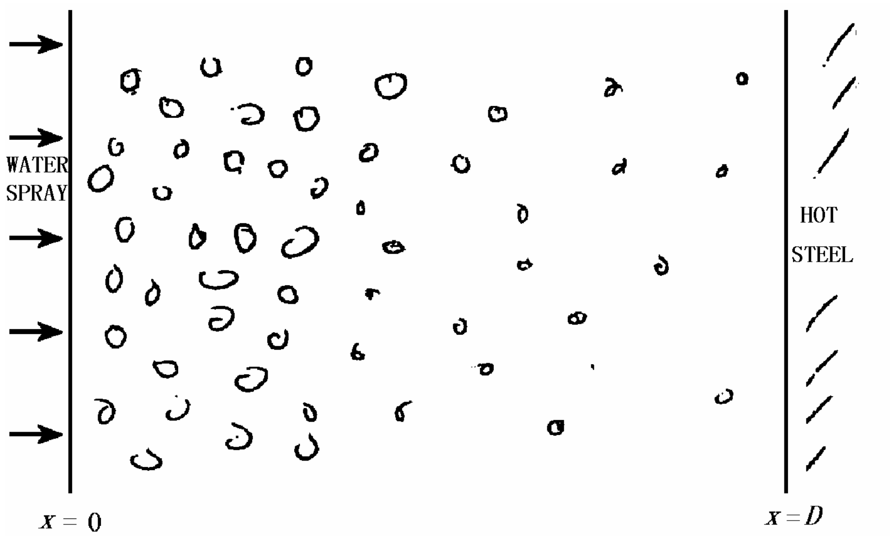

Fig.3 A schematic diagram of spray cooling

The important observation for which a scientific framework is needed that the heat transfer from the steel is a non-monotone function of the steel temperature as in Fig.4.

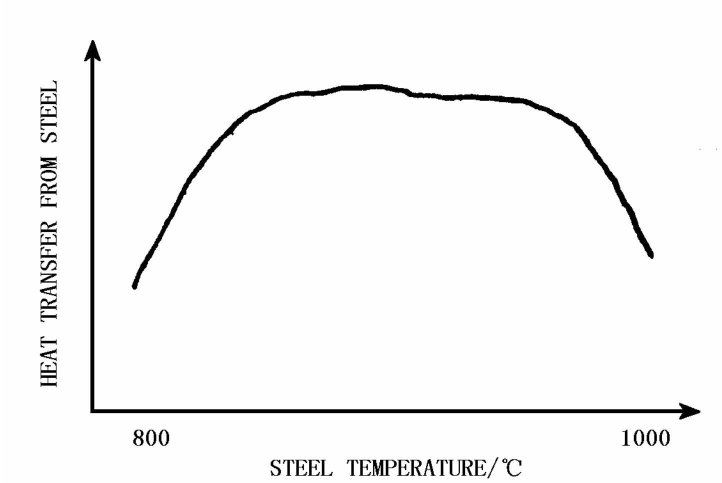

Fig.4 Correlation between steel's surface temperature and heat radiation

In order to understand this phenomenon, we attempt to write down the simplest possible in one spatial dimension, x (as in Fig.3). To do this we do not try to track the fate of every individual droplet but rather work with six averaged_variables, namely the gas and droplet velocities in the x -direction, the gas and droplet temperatures, the gas pressure and the volume fraction Of droplets in the spray. Note that when we work on the length scale of a droplet, the gas and droplet velocities and temperatures must be equal on the droplet surface; however, on a scale large compared to a typical droplet radius but small compared to the spray distance D (Fig.1), the averages of these quantities will not be equal.

We next write down the fundamental physical "conservation laws" for the droplets and surrounding gas. There are two laws for mass conservation (which must account for droplet vaporization), two laws for momentum conservation (which must account For both vaporization and the viscous drag exerted by the gas on the droplets), and one law for energy conservation (which needs to account for the latent heat of vaporization).

The only small complication in this averaged model is that, in order to account for vaporization, we need to incorporate the predictions of a simple "submodel" for a Vaporizing droplet, of radius $r=s(t)$, say (it is easy to see that the volume fraction of droplets is proportional to $s^{1/3}$). This consists in solving a "Stefan" model [2] for the droplet, which has temperature T_v say, as it evaporates into a relatively large gas region at ambient temperature T_g. Note that this requires us to make the implicit assumption that the spray is dilute, so that the droplet separation is much greater than the droplet radius.

Now we come to the easiest, but most important mathematical step of all, nondimensionalisation. When we divide all the physical variables, both dependent and independent, by representative constant values the following key nondimensional parameters emerge. They are:

- The "steel temperature" parameter $R=T_v\ /(T_s-T_v)$ (T_v is the droplet vaporization temperature, T_s is the steel temperature).
- The "droplet size" parameter

$$M = D\, k_g(T_s - T_v) / U \rho_g L s_0^2$$

(k_g is the thermal conductivity of the gas, ρ_g the gas density, U the initial spray velocity, L the droplet latent heat/unit mass and s_0 the initial droplet radius).

- The "spray" parameter $\Gamma = K_g/C_P \rho a_0 UD$ (C_P is the gas specific heat, ρ the droplet density and a_0 the initial volume fraction of droplets).

After some mathematics, the six conservation equations reduce to a first-order ordinary differential equations which is easy to solve numerically. It gives the following dramatically different predictions(Fig.5):

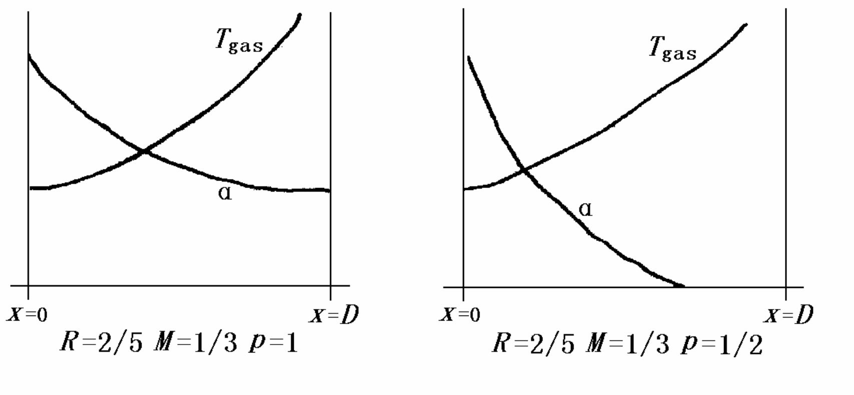

Fig.5 Predictions of droplet size and gas temperature

We see that when we reduce the droplet radius and increase the initial void fraction we can switch from a cooling in which the spray survives to impact the steel, with $\alpha > 0$ at x = D, to one in which "dry-out" occurssw and $\alpha = 0$ in a region near the steel.
We expect optimal cooling efficiency to occur when $\alpha \to 0$ as $x \to D$ and our simple model gives a guide to the operating conditions needed to achieve this optimum.

3. Reactions in a Blast Furnace

A schematic of the complicated heat and mass transfer that can occur in a blast furnace is given in Fig.6.

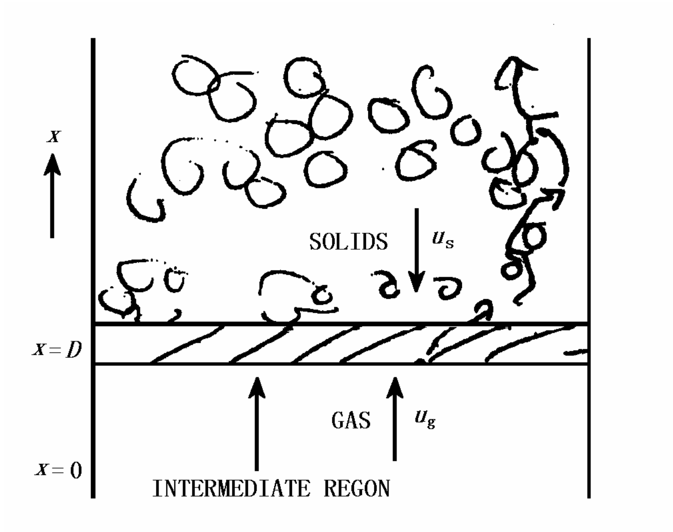

Fig.6 Complicated heat and mass transfer occuring in a blast furnace

An important operating question concerns the stability of the configuration in Fig.6 in which the "intermediate region", where the main chemical reactions occur, is assumed to be horizontal and very thin.
Again, we make the simplest possible one–dimensional model by introducing averaged variables, namely the solid and gas temperatures, the vertical gas velocity μ_g and the gas pressure p (the solid velocity is assumed to be a constant, μ_s).
We again have two equations for energy conservation of gas and solid charge, but only one for mass conservation of the gas, and only one for momentum conservation of the gas. The gas velocity is so fast that the usual Darcy law for porous medium flow does not apply; instead we must use the Ergun equation in which μ and p are related by

$$\mu \mid \mu \mid = -k \partial p / \partial x$$

where k is a constant.
The all-important nondimensionalisation now reveals the following three parameters.

- The gas-flow parameter $\beta = \alpha_g \rho_g c_g u_g / \alpha_s \rho_s c_s u_s$

where α, ρ, c are void fractions, densities and specific heats, with g and s denoting gas and solid, and

$$u_g{}^2 = p_{go} h\, k / \alpha_s \rho_s c_s u_{s,}$$

Where p_{go} is the gas pressure drop across the charge, and h the depth of the charge.

- The gas temperature parameter $\gamma = T_{go}/(T_m - T_a)$

(where T_{go} is the gas inlet temperature, T_m the reaction temperature and T_a the ambient temperature).

- The heat of reaction parameter $\delta = \Delta H \alpha_c / \rho_s c_s (T_m - T_a)$ (when ΔH is the heat of reaction and α_c the volume fraction of reacting solids).

The model is so simple that we now only have an algebraic equation to solve for the position $x = d$ of the intermediate region. The dramatic prediction is now that, for given values of the parameters there are either two solutions for d or none as in Fig.7.

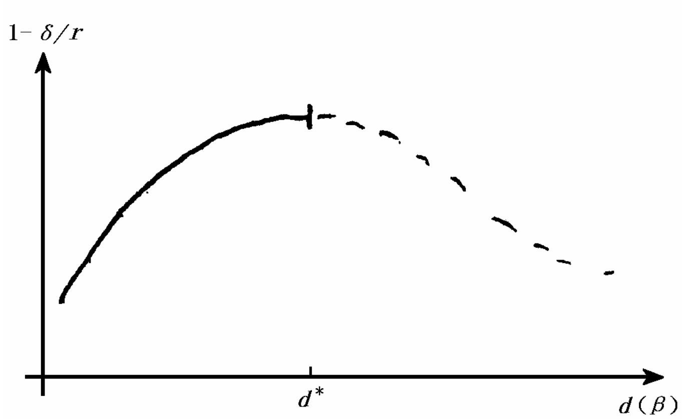

Fig.7 Prediction for position of the intermediate region

Moreover, after more work involving the introduction of time - derivatives into the conservation laws, we can show that, for $d>d^*$ the dotted branch of the response in Fig.8 is unstable to one −dimensional perturbations, the continuous branch for $d>d^*$ being stable.

This analysis strongly suggest that we consider the two−dimensional stability of our steady state solution. This involves introducing the two −dimensional Ergun equation

And the analysis eventually enables us to see that short "waves" (Fig.8a) are stable if U_s is small enough but that long waves (Fig.8b) are always unstable. Moreover, the model does predict how the time scale of the instability depends on the parameter $\sqrt{d} / \beta$.

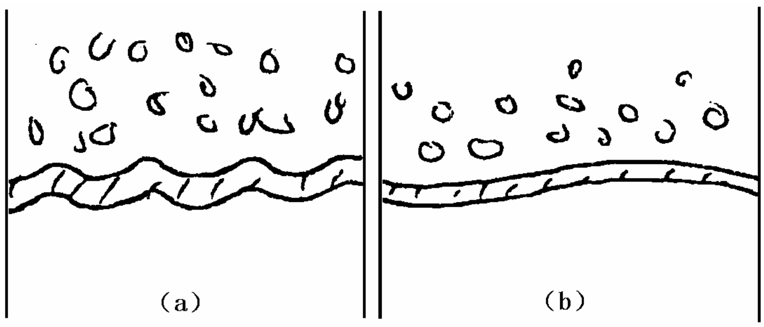

Fig.8 Correlation between wave length and stability

4.Conclusion

Although this article has emphasised the steel industry, its most important themes apply to all differential equation models for industrial processes. These themes are

- To begin with the simplest model that incorporates the fewest mechanisms. It is easier to add to a simple model than to begin with a more complicated one.
- To nondimensionalise the model so as to reveal the crucial nondimensional parameters.

- To create a scientific framework in the space of these nondimensional parameters.

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